Int. J. Heat Mass Transfer. Vol. 26, No. 11, pp. 1685-1693, 1983
Printed in Great Britain

0017-9310/83 $3.00 + 0.00
Pergamon Press Ltd.

AN IMPROVED ROOT-FINDING PROCEDURE FOR USE IN CALCULATING TRANSIENT HEAT FLOW THROUGH MULTILAYERED SLABS

DOUGLAS C. HITTLE and RICHARD BISHOP*

Energy Systems Division, U.S. Army Construction Engineering Research Laboratory, P.O. Box 4005, Champaign, IL 61820, U.S.A.

(*Received* 12 *February* 1982 *and in final form* 14 *March* 1983)

Abstract—So-called 'response factors' are the flux at the inside and outside surfaces of a one-dimensional multilayered slab caused by unit triangular temperature pulses alternately applied to the inside and outside surfaces while holding the opposite surface at constant temperature. Most recent techniques for finding response factors involve a numerical search for the roots of the characteristic equation of the Laplace transformed solution to the heat conduction equation. Once these poles are known, residue-calculus is used to find the inverse transform which yields response factors. This paper examines the behavior of the characteristic equation and related equations and presents an improved root-finding procedure which allows response factors to be calculated efficiently.

NOMENCLATURE

C	thermal capacitance [J m^{-2} K^{-1}]
c_p	specific heat [J kg^{-1} K^{-1}]
F	solution matrix
K	coefficient matrix
k	conductivity [W m^{-1} K^{-1}]
l	thickness [m]
M	transmission matrix
N	integer counter
q	heat flux [W m^{-2}]
R	thermal resistance [m^2 W^{-1} K^{-1}]
s	Laplace transform variable
T	temperature [°C or K]
t	time [s]
U	vector
x	location [m].

Greek symbols

α	thermal diffusivity, $k/\rho c_p$
β	the negative of the Laplace parameter, s
Γ_A, Γ_B	families of curves of zeros of $A(x,\beta)$ and $B(x,\beta)$
ρ	density [kg m^{-3}].

Subscripts

i	ith layer.

INTRODUCTION

THE USE of so-called 'response-factors' [1, 2] to solve transient heat conduction problems in multilayered slabs has increased with the development of detailed building energy analysis computer programs [3-6]. The technique is particularly important in faithfully characterizing the 'dynamic' response of multilayered walls, roofs, and floors, although it need not be limited to this application.

An important improvement in the procedure used for calculating response factors is presented in this paper. This new procedure improves the reliability of the method without sacrificing computational efficiency.

HEAT CONDUCTION THROUGH MULTILAYERED SLABS

Heat conduction through a one-dimensional homogeneous slab is governed by the following second-order partial differential equation:

$$\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T(x,t)}{\partial t}, \tag{1}$$

where T is the temperature at position x and time t, α is the thermal diffusivity, $\alpha = k/\rho c_p$, k is the thermal conductivity [W m^{-1} K^{-1}], ρ is density [kg m^{-3}], and c_p is the specific heat [J kg^{-1} K^{-1}]. The heat flux at any position x and time t is given by:

$$q(x,t) = -k\frac{\partial T(x,t)}{\partial x}. \tag{2}$$

In both the above relations, k, ρ, and c_p were assumed to be constant (we will later relax this assumption to allow k, ρ, and c_p to vary spatially, i.e. with x).

The response factors are defined to be the time series of fluxes $q(0, i\Delta t)$, $q(l, i\Delta t)$, $i = 1, 2, 3, \ldots$ which result when unit triangular temperature pulses with base $2\Delta t$ are applied first on the surface $x = 0$ and then on the surface $x = l$, while the opposite surface is held at the initial temperature. Due to the fact that the differential equations are linear and autonomous, the responses to a sum of pulses of various sizes and starting times can be calculated by superposition of the standard responses appropriately shifted and weighted. This allows the fluxes to be calculated exactly for those surface temperature profiles which are obtained by linear interpolation between the values at multiples of Δt. The response factors can be calculated to a specified accuracy once and for all, and then any errors in determining the response to given surface temperature profiles are due to the approximation by such

* Present address: Department of Mathematics, University of Illinois, Urbana, IL 61801, U.S.A.

trapezoidal profiles. There are no errors arising from making approximations to spacially dependent temperatures such as there would be in lumped parameter or finite difference techniques. One no longer has to worry about the stability of numerical methods and their convergence to genuine solutions. The error to the method can be simply visualized as the response to the difference between the actual surface temperature profiles and their trapezoidal approximations. Since temperatures are generally measured at discrete times, the linear interpolation is one of the more reasonable estimates for them anyway.

In practice, the fluxes are calculated recursively in terms of previous temperatures and fluxes. The coefficients of the recursive relations are determined from the response factors in such a way as to minimize the number of historical items required.

The analytical calculation of response factors is conveniently done by using Laplace transforms on the time variable. Because the activating triangular pulses have initial value zero, the time derivative is expressed by multiplication by the frequency-domain variable s. The physical derivation of equation (1) is through the equivalent first-order system in the pair $T(x,t)$ and $q(x,t)$. Indeed, it is the first-order system which remains valid when one considers more general situations in which the coefficients k and α become functions of x, as we do in the proof of the root-separation theorem below. The Laplace transform of this system, under the assumption that $T(x,0) = q(x,0) = 0$, is

$$\frac{dT(x,s)}{dx} = -\frac{1}{k}q(x,s), \tag{3}$$

$$\frac{dq(x,s)}{dx} = -\frac{k}{\alpha}sT(x,s). \tag{4}$$

If we assume that the wall is made of layers on which k and α are constant, then in each layer, the general solution of equations (3) and (4) is given in terms of hyperbolic functions of $l_i\sqrt{(s/\alpha)}$, where l_i are the widths of the layers. If we specify T and q on one face of a layer, then they are determined throughout the layer, and, in particular, on the other face. Since it is physically reasonable, we require T and q to be continuous across layer boundaries, so that the values on the second face should be used as initial values for the next layer. Thus, any initial values at one face of the slab are transmitted layer-by-layer across the wall until we get values on the other face. Because the differential equations are linear and homogeneous, the transmission of initial values is a linear transformation, given by matrix multiplication. That is, we may claim

$$\begin{bmatrix} T(0,s) \\ q(0,s) \end{bmatrix} = \begin{bmatrix} A(s) & B(s) \\ C(s) & D(s) \end{bmatrix} \begin{bmatrix} T(l,s) \\ q(l,s) \end{bmatrix}, \tag{5}$$

for arbitrary initial values $T(l,s)$ and $q(l,s)$. This defines the transmission matrix

$$M(s) = \begin{bmatrix} A(s) & B(s) \\ C(s) & D(s) \end{bmatrix}.$$

The reason for choosing the transmission in this apparently backward way is so that $M(s)$ can be realized as a product from left to right of transmission matrices for the individual layers:

$$M(s) = M_1(s)M_2(s),\ldots,M_n(s). \tag{6}$$

By solving the differential equations explicitly, we find

$$M_i(s) = \begin{bmatrix} \cosh[l_i\sqrt{(s/\alpha_i)}] & [1/k_i\sqrt{(s/\alpha_i)}]\sinh[l_i\sqrt{(s/\alpha_i)}] \\ [k_i\sqrt{(s/\alpha_i)}]\sinh[l_i\sqrt{(s/\alpha_i)}] & \cosh[l_i\sqrt{(s/\alpha_i)}] \end{bmatrix}, \tag{7}$$

where k_i and α_i are the thermal conductivity and thermal diffusivity of the layer, respectively. The matrix entries are called transfer functions.

Our goal is to relate temperature inputs to flux outputs, not, as equation (5) might seem to indicate, to relate a temperature and flux on one side of the slab to that on the other. We view equation (7) as a necessary condition on the transforms of the functions involved in the problem at hand. We continue with the customary procedures of Laplace transform technique: solve for the transforms of the outputs in terms of those of the inputs, calculate the inverse transforms, and verify that the infinite series so obtained converge to actual solutions of the original differential equations and boundary conditions. To describe all of these procedures in detail would make a rather lengthy technical treatise, but would involve only modifications of matters adequately covered in elementary tests on transform methods. The solution for flux transforms is given by

$$\begin{bmatrix} q(0,s) \\ q(l,s) \end{bmatrix} = \begin{bmatrix} \dfrac{D(s)}{B(s)} & -\dfrac{1}{B(s)} \\ \dfrac{1}{B(s)} & -\dfrac{A(s)}{B(s)} \end{bmatrix} \begin{bmatrix} T(0,s) \\ T(l,s) \end{bmatrix}. \tag{8}$$

To calculate response factors, the technique of residue-calculus inversion of the transforms is used. The poles involved come from whatever we choose for $T(0,s)$ or $T(l,s)$ and from the zeros of $B(s)$. In the course of our proof below that the zeros of $A(s)$ and $B(s)$ separate each other, we show that the zeros of $B(s)$ are negative and simple. When $T(s)$ is a triangular pulse transform, its only pole is a double pole at $s = 0$. Note that due to the form of equation (8), the two cross response factor series, relating temperature on one side to flux on the other, are negatives of each other.

It is convenient to set $s = -\beta$, so that for the purpose of calculating roots, β is a positive real number. It is also convenient to use thermodynamic resistance and capacitance as the characteristic properties of each layer. These are defined to be $R_i = l_i/k_i$ and $C_i = l_i\rho_i c_{pi}$. Using this notation, the transmission matrix for a layer becomes

$$\begin{bmatrix} \cos\sqrt{(\beta RC)} & [R/\sqrt{(\beta RC)}]\sin\sqrt{(\beta RC)} \\ -[\sqrt{(\beta RC)}/R]\sin\sqrt{(\beta RC)} & \cos\sqrt{(\beta RC)} \end{bmatrix}.$$

When a layer has negligible heat capacity compared to its resistance, it is a reasonable simplification to take the limit as $C \to 0$, so that the matrix is

$$\begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix}.$$

For cases involving three or more layers, elements of the transmission matrix are complicated products and sums of the transfer functions for each layer [see equation (12)]. Therefore, in practice, the roots of $B(\beta)$ are found by numerical search. The procedure used was to scan an interval between $0 < \beta \leqslant \beta_{max}$ in steps, evaluating $B(\beta)$ at each step (see Fig. 1). The occurrence of a sign change in $B(\beta)$ between steps indicated that a root had been bracketed. Figure 1 shows $B(\beta)$ as a function of β for a simple one-layer case. The change in sign of $B(\beta)$ between β_2 and β_3 brackets the first root of $B(\beta)$ (the step size has been exaggerated for the purpose of illustration).

Once a root was bracketed, a secant-method, root-finding procedure was used to pinpoint the value of β where $B(\beta) = 0$. (See Hittle [7] for a more detailed discussion of response factor methods.)

For certain slabs, the roots of $B(\beta)$ occur in pairs which are extremely close together (of the order of 10^{-8} apart). Figure 2 shows two such cases, both of which can be realized by heavy concrete layers with an intermediate layer of insulation. The deficiency of the above procedure, when applied to cases like those of Fig. 2, was that it would step over pairs of roots without detecting a sign change in $B(\beta)$. The fact that roots had been missed was detected by an energy conservation check, but missed pairs of roots could not be found without resorting to time-consuming scans with extremely small steps.

A new procedure for finding the roots of $B(\beta)$ has been developed based on the discovery that the roots of the transfer function $B(\beta)$ are separated by roots of the transfer function $A(\beta)$. This procedure will now be presented; in the next section, we give a proof of the root-separation theorem. It has been implemented in a research version of BLAST [5] by G. Walton of NBS who has made tests of its speed and efficiency which verify that it does indeed give a substantial improvement [8].

We first note that, for each step along the β axis, $A(\beta)$ was being evaluated each time $B(\beta)$ was evaluated. This was a natural consequence of the matrix multiplications necessary to calculate $B(\beta)$. We begin the new search algorithm in the same way as before—stepping along the β axis looking for changes in sign in $B(\beta)$. However, we also keep track of the sign of $A(\beta)$ using a counter N, which is the number of sign changes in $A(\beta)$ since a root of $B(\beta)$ was found. When $N = 2$ is detected, it is known that two roots of $B(\beta)$ have been jumped over. When this occurs, a secant-method, root-finding algorithm is used to locate the root of $A(\beta)$, but only with enough accuracy to bracket the missing roots of $B(\beta)$.

Figure 3 will help make this procedure clear. Figure 3(a) and (b) are sample plots of $B(\beta)$ and $A(\beta)$ for the same multilayer slabs as in Figs. 2(a) and (b). The scales have changed to accommodate the plotting of $A(\beta)$. Figures 3(a) and (b) clearly show how the roots of $A(\beta)$ separate the roots of $B(\beta)$.

Figure 3(c) is an exaggerated view of one of the 'bad' regions of the locus of $B(\beta)$, showing how the new search algorithm isolates the roots of $B(\beta)$. We denote as β_1 the value of β at which we evaluate $A(\beta)$ and $B(\beta)$ prior to the occurrence of the pair of roots of $B(\beta)$. β_2 is one step further along the axis. By referring to Fig. 3(b), we see that there has been one sign change in $A(\beta)$ between the last root of $B(\beta)$ and β_1; hence, $N = 1$ at β_1. Another

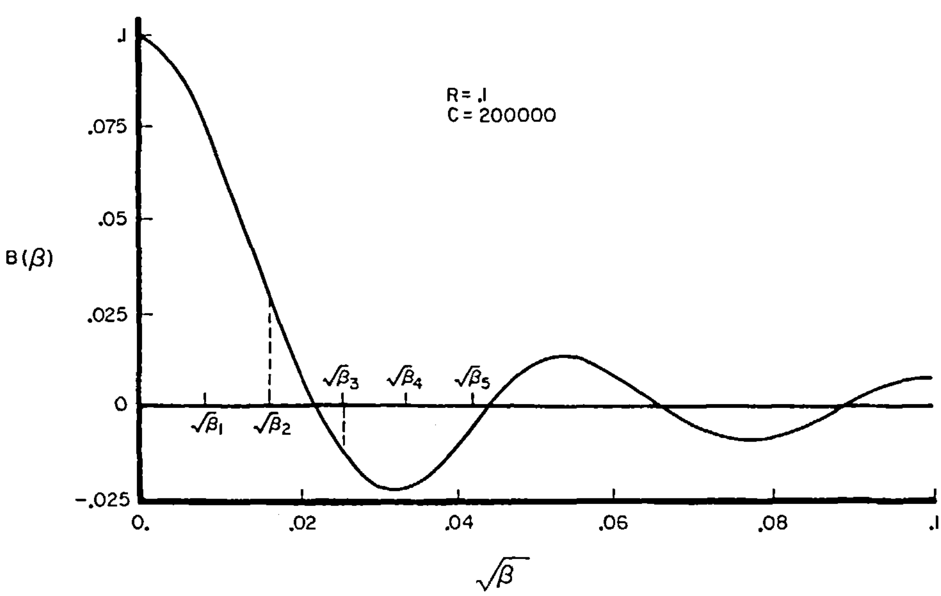

FIG. 1. Locus of $B(\beta)$ for a homogeneous slab.

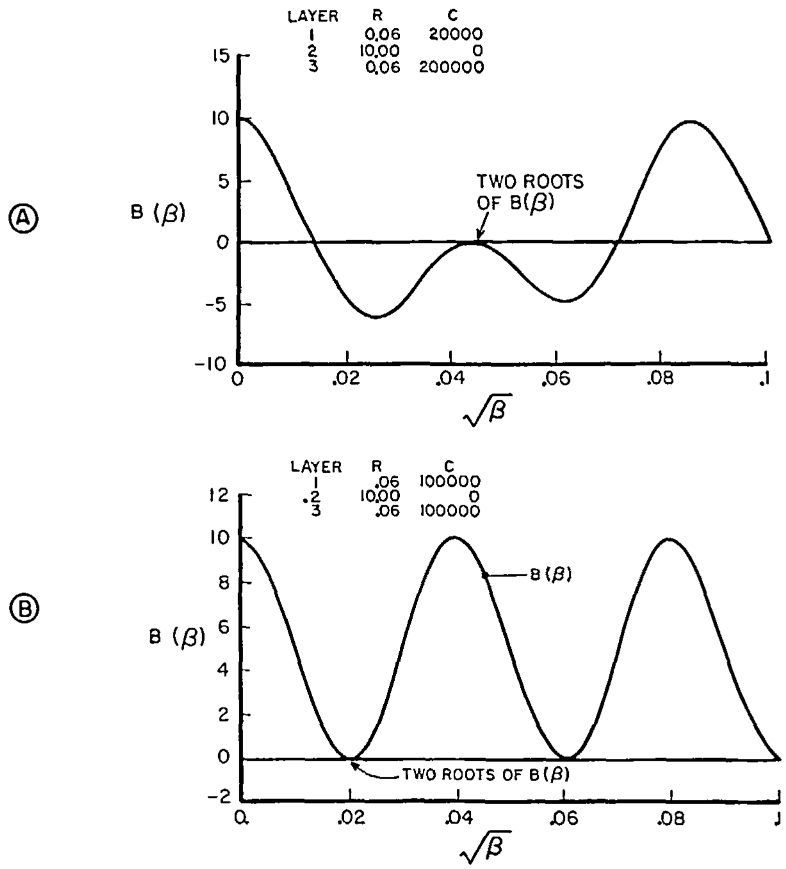

FIG. 2. Locus of $B(\beta)$ for selected three-layer slabs.

sign change occurs in $A(\beta)$ between β_1 and β_2; hence, $N = 2$ at β_2, and we know we have missed two roots of $B(\beta)$. We now begin the secant-method search for the root of $A(\beta)$, using β_1 and β_2 as starting values. At each iteration of the search, we check the sign of $B(\beta)$. As soon as $B(\beta)$ changes sign [at β' in Fig. 3(c), for example], we can stop searching for the root of $A(\beta)$, since we have bracketed the roots of $B(\beta)$. We need not find the root of $A(\beta)$ exactly. We now invoke the secant-method root-finder two more times using β_1 and β', and β' and β_2 as starting points to find the desired roots of $B(\beta)$.

A final important step is to record the sign of $A(\beta)$ at the right-most root of $B(\beta)$, at β'' in Fig. 3(c), and reset N to zero. We then proceed with our stepwise search for more roots, beginning at β_2.

There are a number of other possible relationships between the search points (denoted as β_1 and β_2) and the zero-crossings of $A(\beta)$ and $B(\beta)$, so that the handling of N requires some care. Figure 4 shows an exaggerated view of the possibilities and explains the procedures used in each case.

PROOF OF THE SEPARATION OF ROOTS

To prove that the zeros of $A(\beta)$ and $B(\beta)$ separate each other, we consider extensions of these functions to the x–β plane, denoted $A(x,\beta)$ $B(x,\beta)$. We gain some generality with no additional effort by allowing the physical quantities of the system to be piecewise differentiable functions of x, rather than piecewise constant, since they are usually taken for multilayered slabs. It is convenient to let $r(x) = 1/k(x)$, where $k(x)$ is the conductivity at x, and $c(x) = \rho(x)c_p(x)$, where $\rho(x)$ and $c_p(x)$ are the density and specific heat at x. Then we can obtain overall resistances and capacitances by integrating $r(x)$ and $c(x)$.

The transformed differential equations—equations (3) and (4)—can be written in matrix form in terms of the column

$$U(x,\beta) = \begin{bmatrix} T(x,\beta) \\ q(x,\beta) \end{bmatrix}$$

and the coefficient matrix

$$K(x,\beta) = \begin{bmatrix} 0 & -r(x) \\ \beta c(x) & 0 \end{bmatrix}$$

as

$$\frac{dU(x,\beta)}{dx} = K(x,\beta)U(x,\beta). \tag{9}$$

The general solution of equation (9) is commonly

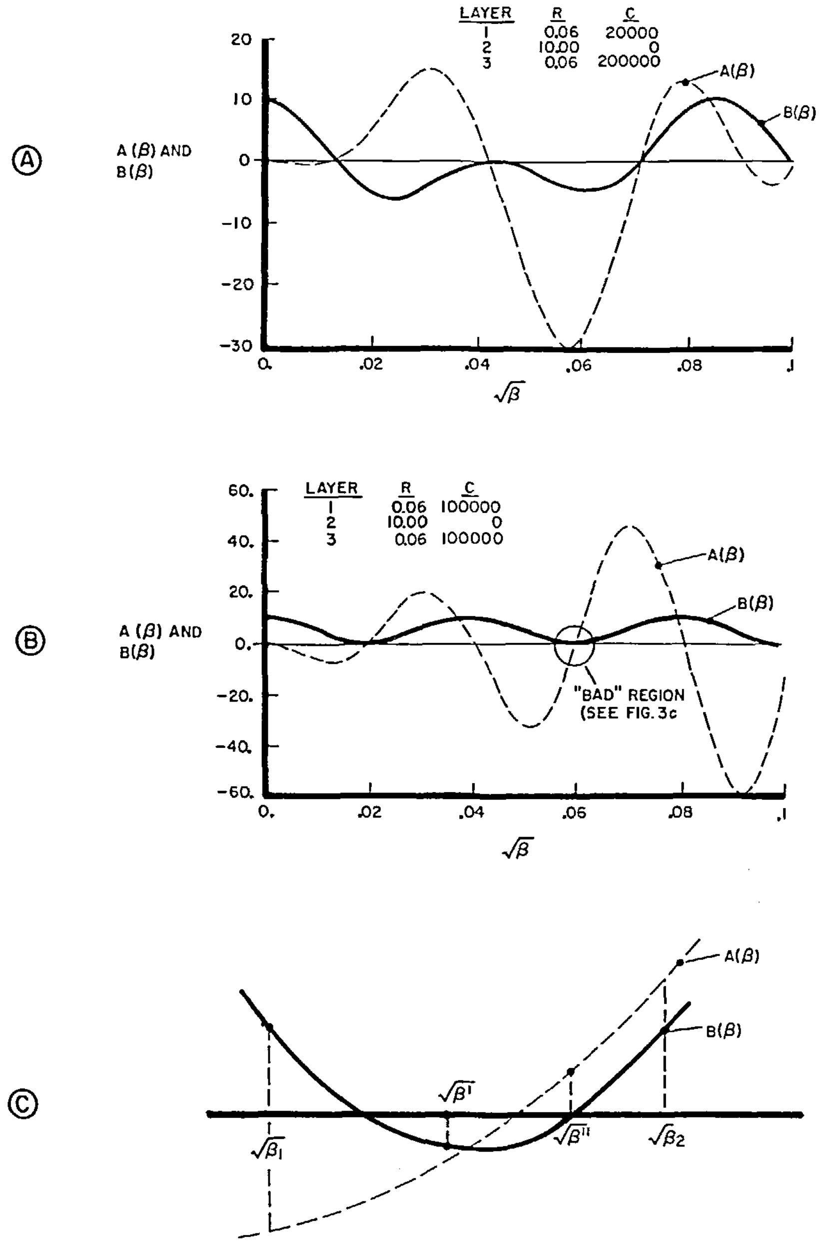

FIG. 3. Locus of $A(\beta)$ and $B(\beta)$ for selected three-layer slabs.

expressed in terms of the fundamental 2×2 matrix solution $F(x, \beta)$, defined as the solution of the initial value problem

$$F(0, \beta) = I, \quad \frac{dF}{dx} = KF. \tag{10}$$

Then the general solution for a column vector $U(x, \beta)$ is given in terms of the initial value $U(0, \beta)$ by

$$U(x, \beta) = F(x, \beta) U(0, \beta). \tag{11}$$

It is a standard theorem about matrix differential equations, related to Abel's theorem on the Wronskian, that the determinant of a solution satisfies a DE too, with the trace of the coefficient matrix as its coefficient:

$$\frac{d}{dx} \det F = (trK) \det F.$$

In the case at hand, $trK = 0$ and we conclude that $\det F = 1$ constantly.

We let the inverse matrix of F be M, so that from equation (11)

$$U(0, \beta) = M(x, \beta) U(x, \beta). \tag{12}$$

If we compare this with equation (5), which defined the matrix, we called $M(s)$, we conclude that $M(s) = M(l, \beta)$, because in both cases, the equation must hold for

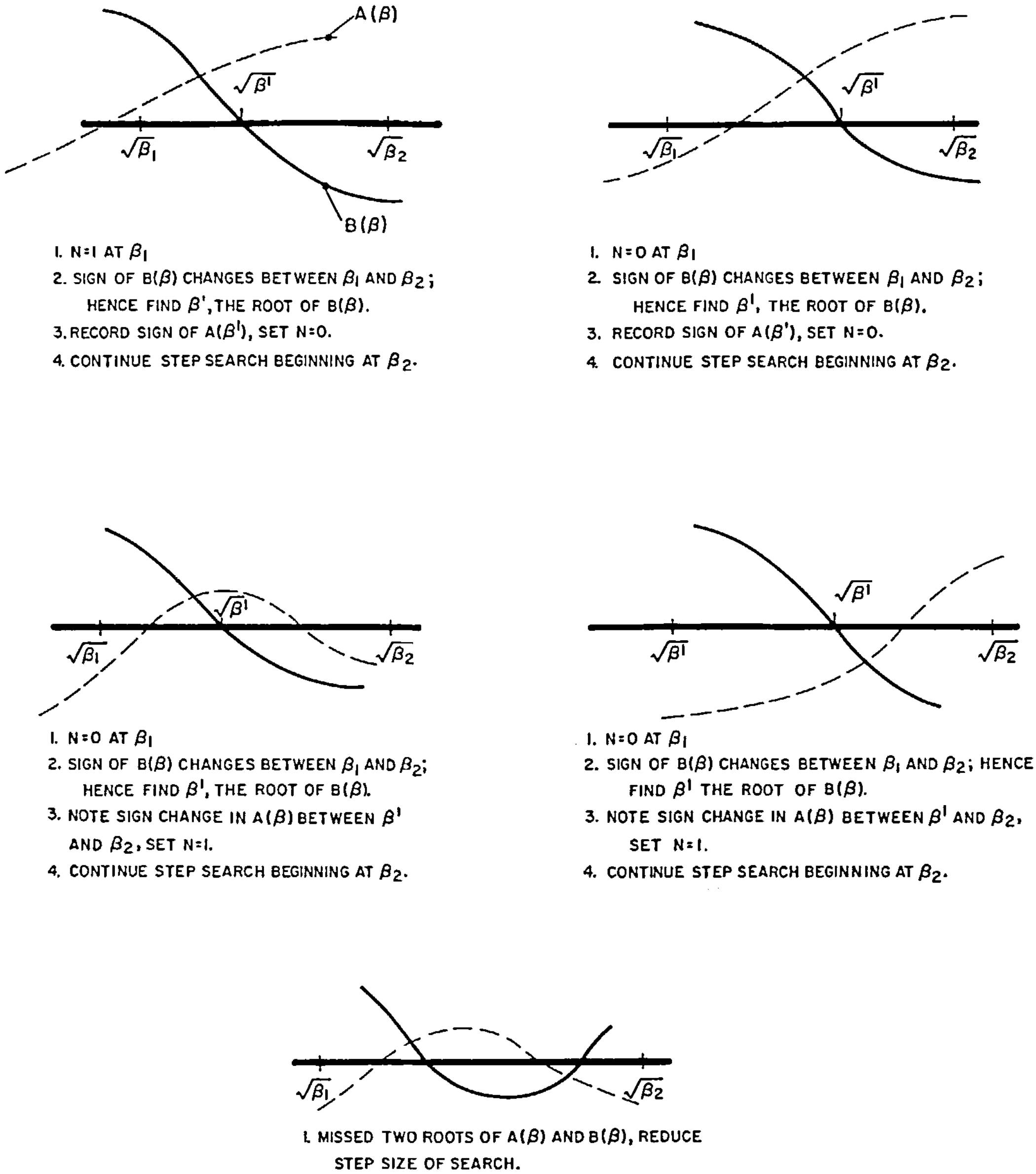

FIG. 4. Possible relationships between roots of $A(\beta)$ and $B(\beta)$.

arbitrary values of the column $U(l, \beta)$. Hence, denoting the entries of $M(x, \beta)$ by $A(x, \beta)$, $B(x, \beta)$, $C(x, \beta)$, and $D(x, \beta)$, we get the desired extensions of $A(\beta)$ and $B(\beta)$ to the x–β plane. Since $\det M = 1$, the formula for F is

$$F(x, \beta) = \begin{bmatrix} D(x, \beta) & -B(x, \beta) \\ -C(x, \beta) & A(x, \beta) \end{bmatrix}. \tag{13}$$

We are now in a position to prove that the zeros of $B(\beta)$ are positive (s is negative). Let β be any complex number and suppose we have $B(\beta) = 0$. Let $f(x) = -B(x, \beta)$, $g(x) = A(x, \beta)$, so that these are the possibly-complex-valued functions which form the second column of $F(x, \beta)$. By equation (10) we have $f(0) = 0$, $f'(x) = -r(x)g(x)$, and $g'(x) = \beta c(x)f(x)$. We have assumed that $f(l) = 0$. Using an overbar to denote complex conjugates, we have

$$\beta \int_0^l c(x)f(x)\overline{f(x)}\,dx = \int_0^l g'(x)\overline{f(x)}\,dx$$

$$= g(l)\overline{f(l)} - g(0)\overline{f(0)} - \int_0^l \overline{f'(x)}g(x)\,dx$$

$$= \int_0^l r(x)\overline{g(x)}g(x)\,dx. \tag{14}$$

This shows that β is the quotient of the two positive integrals appearing in equation (14).

If one row of a matrix is zero, then the determinant is zero. Since $\det M = 1$, we conclude that $A(x, \beta)$ and $B(x, \beta)$ cannot be zero simultaneously. If we were to assume that $r(x)$ and $c(x)$ are continuous and never zero, then it would follow immediately from Rolle's theorem and equation (10) that as x varies, for β constant, the zeros of $B(x, \beta)$ and $A(x, \beta)$ alternate, separating each other. When we allow $c(x)$ to be zero on a whole interval, then it is possible for $A(x, \beta)$ to vanish on such an interval too. However, if we count such an interval of zeros of $A(x, \beta)$ as just a special sort of single zero, then it is still true that the zeros of $B(x, \beta)$ and $A(x, \beta)$ alternate. The proof is a technicality which is designed to get around the use of Rolle's theorem, and we will not give all the details here. The idea is that between two zeros of, say, $B(x)$, $B(x)$ must first rise, then fall, or vice versa so that its derivative must have points in such an interval where it is opposite in sign. By the differential equation (10), the other function $A(x)$ must also change sign, so that it is zero some place in between. Figure 5 shows typical behavior of $A(x)$ and $B(x)$ with $r(x)$ and $c(x)$ constant on each of three segments, with $c(x) = 0$ in the middle.

Thus, the zeros of $A(x, \beta)$ and $B(x, \beta)$ separate each other when β is constant. We must now show that they separate each other when x is constant and β varies. We define families of curves in the x–β plane called Γ_A and Γ_B, where $A(x, \beta) = 0$ and $B(x, \beta) = 0$, respectively. The separation properties these curves have in the x-direction would do no good if the curves had a Z shape as pictured in Fig. 6. But this behavior is not possible because the curves Γ_A and Γ_B always have nonpositive slope. The fact that the zeros of $B(\beta)$ are simple is a consequence, since the vertical line $x = l$ must meet the curves Γ_B with simple crossings, not tangentially.

We calculate the slopes of Γ_A and Γ_B by using implicit differentiation on the defining equations $A(x, \beta) = 0$ and $B(x, \beta) = 0$. Thus, $d\beta/dx = -A_x/A_\beta$ on Γ_A, and $d\beta/dx = -B_x/B_\beta$ on Γ_B, where the subscripts denote partial derivatives. To get the necessary information about A_β and B_β, we use the fact that they must satisfy differential equations and initial conditions which we get from differentiating equation (10) with respect to β. If we let $G(x, \beta) = F_\beta(x, \beta)$, then the DE satisfied by G is

$$\frac{dG}{dx} = KG + K_\beta F. \tag{15}$$

The method of variation of parameters leads to the following integral expressions for A_β and B_β, which can be verified as solutions directly

$$A_\beta(x, \beta) = -A(x)\int_0^x c(u)B(u)D(u)\, du + C(x)\int_0^x c(u)B(u)^2\, du,$$

$$B_\beta(x, \beta) = -B(x)\int_0^x c(u)B(u)D(u)\, du + D(x)\int_0^x c(u)B(u)^2\, du.$$

In these formulas we have abbreviated $A(x, \beta)$ and $B(x, \beta)$ to $A(x)$ and $B(x)$. Along the curves Γ_A, we have $A(x) = 0$ and $C(x) = 1/B(x)$, so that the first integral drops out and A_β reduces to $A_\beta(x, \beta) = -H(x)/B(x)$, where $H(x)$ is the positive integral

$$H(x) = \int_0^x c(u)B(u)^2\, du.$$

Then we have $d\beta/dx = -\beta c(x)B(x)^2/H(x) \leqslant 0$. Similarly, along the curves Γ_B, we have $D(x) = 1/A(x)$ and $d\beta/dx = -r(x)A(x)^2/H(x) \leqslant 0$.

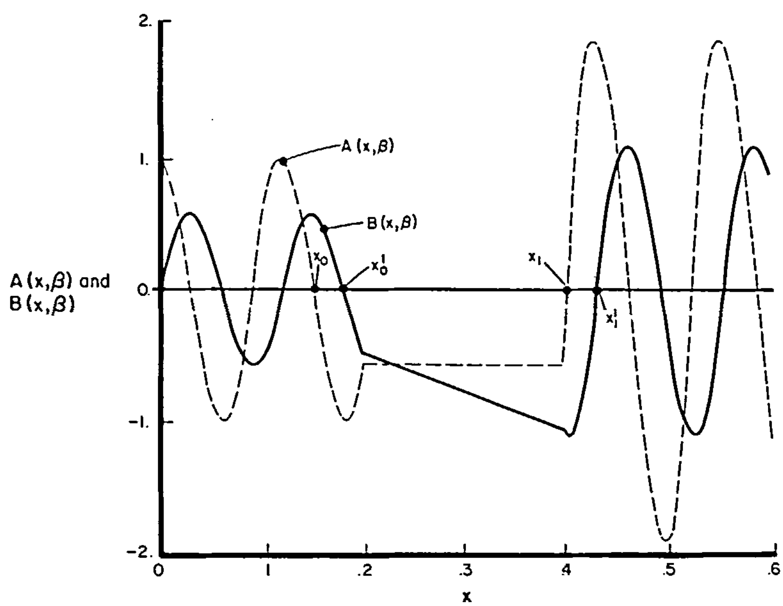

FIG. 5. Locus of $A(x, \beta)$ and $B(x, \beta)$ for constant β.

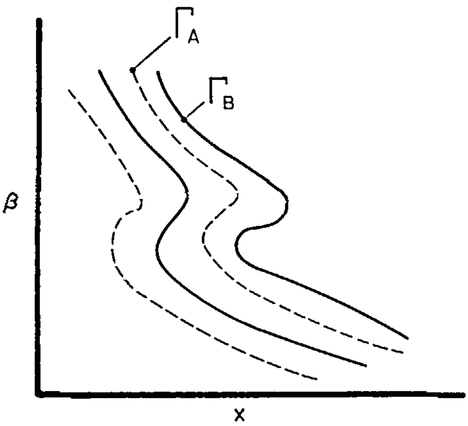

FIG. 6. Impossible Z-shape for Γ_A and Γ_B curves.

The picture which has evolved is this (see Fig. 7). Each connected curve of the family of Γ_A and/or Γ_B divides the strip $0 < x \leqslant l$, $0 < \beta$ into an upper and a lower part. In the band between two successive Γ_B there is just one Γ_A curve and vice versa. Since these alternating curves terminate on the vertical line at $x = l$, roots of $B(l, \beta) = 0$ are separated by roots of $A(l, \beta) = 0$ and vice versa.

CONCLUSIONS

A significant improvement to the procedure for finding the roots of the characteristic equation used in calculating response factors has been developed. The improvement eliminates the need for an extremely fine step size when numerically searching for roots and ensures that roots will not be missed. The new procedure allows the calculation of response factors for certain multilayered slabs which previously could only be found with unrealistically small search increments. Computational efficiency has been retained while improving reliability.

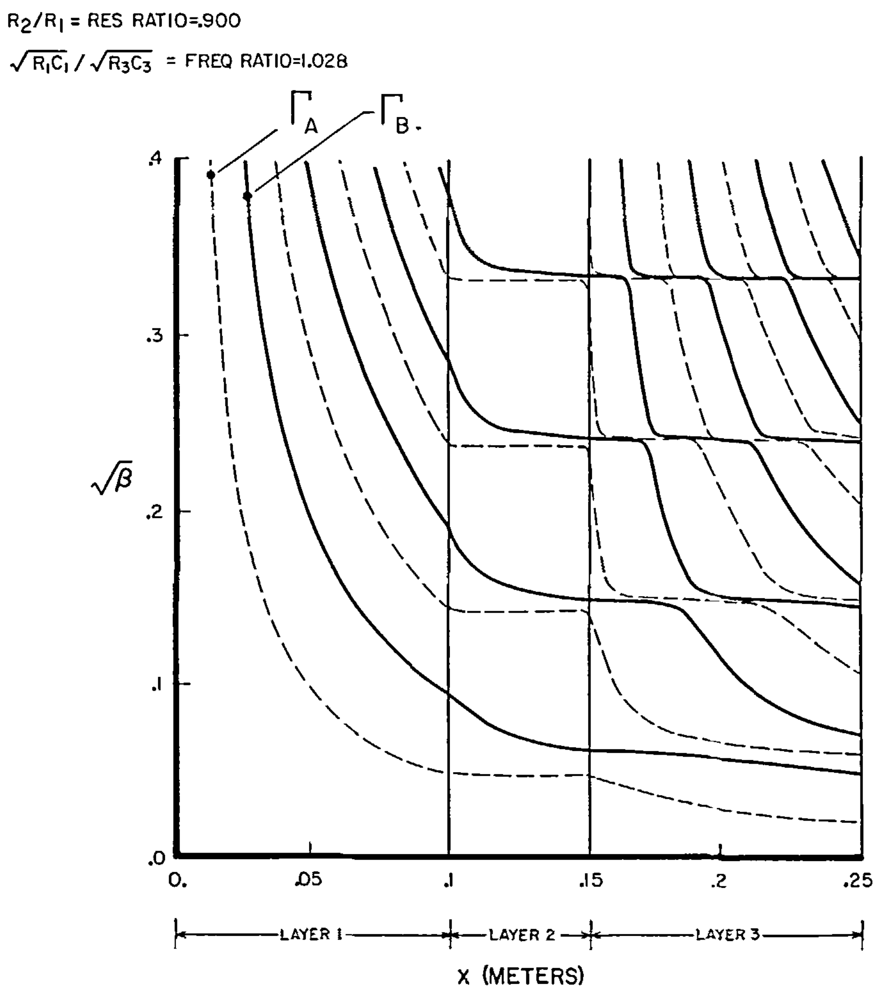

FIG. 7. Γ_A and Γ_B as functions of x and β.

REFERENCES

1. D. G. Stephenson and G. P. Mitalas, Cooling load calculations by thermal response factor method, in *ASHRAE Transactions*, Vol. 73, Part 1, pp. III.1.1–1.7. American Society of Heating, Refrigerating, and Air-conditioning Engineers, Inc. (1967).
2. G. P. Mitalas and D. G. Stephenson, Room thermal response factors, in *ASHRAE Transactions*, Vol. 73, Part 1, pp. III.2.1–2.5. American Society of Heating, Refrigerating, and Air-conditioning Engineers, Inc. (1967).
3. Procedure for determining heating and cooling loads for computerizing energy calculations. Algorithms for building heat transfer subroutines, Energy Calculations 1, ASHRAE Task Group on Energy Requirements for Heating and Cooling of Buildings, American Society of Heating, Refrigerating, and Air-conditioning Engineers, Inc., New York (1975).
4. T. Kusuda, NBSLD, the computer program for heating and cooling loads in buildings, National Bureau of Standards, Washington, DC (1979).
5. D. C. Hittle, Building loads analysis and system thermodynamics program (BLAST) users manual, Version 2, Technical Report E-153, U.S. Army Construction Engineering Research Laboratory, Champaign, Illinois (1979).
6. DOE-2 reference manual (Version 2.1) (edited by D. A. York and E. F. Tucker), Report LA-7689-M, Version 2.1, Los Alamos Scientific Laboratory, Los Alamos, New Mexico (May 1980).
7. D. C. Hittle, Calculating building heating and cooling loads using the frequency response of multilayered slabs, Ph.D. thesis, University of Illinois, Urbana, Illinois (1981) and Technical Manuscript E-169, U.S. Army Construction Engineering Research Laboratory, Champaign, Illinois (1981).
8. Personal communications with G. N. Walton, National Bureau of Standards, Center for Building Technology, Washington, DC (March 1982).

UNE PROCEDURE DE RECHERCHE DES RACINES POUR LE CALCUL DES FLUX THERMIQUES A TRAVERS DES PLAQUES A PLUSIEURS COUCHES

Résumé—Des "facteurs de réponse" sont les flux aux surfaces d'entrée et de sortie d'une plaque multicouche monodimensionnelle causés par des impulsions triangulaires de température appliquées alternativement aux surfaces d'entrée et de sortie, en maintenant la surface opposée à une température constante. Des techniques récentes pour trouver les facteurs de réponse passent par une recherche numérique des racines de l'équation caractéristique de la transformée de Laplace de la solution de l'équation de la conduction thermique. Une fois que ces pôles sont connus, le calcul des résidus est utilisé pour trouver la transformée inverse qui conduit aux facteurs de réponse. Cet article examine le comportement de l'équation caractéristique et des équations associées et il présente une procédure améliorée de recherche des racines qui donne rapidement les facteurs de réponse.

EINE VERBESSERTE METHODE ZUM AUFFINDEN VON LÖSUNGEN BEI DER INSTATIONÄREN WÄRMELEITUNG IN VIELSCHICHTIGEN PLATTEN

Zusammenfassung—Der sogenannte Reaktionsfaktor ist die Wärmestromdichte an der inneren und äußeren Oberfläche einer eindimensionalen vielschichtigen Platte, die durch dreiecksförmige Einheits-temperatursprünge an der inneren und äußeren Oberfläche hervorgerufen wird, während man die gegenüberliegende Fläche auf konstanter Temperatúr hält. Die meisten herkömmlichen Methoden zum Bestimmen von Reaktionsfaktoren erfordern numerisches Suchen der Wurzeln der charakteristischen Gleichung der Laplace-transformierten Lösung der Wärmeleitungsgleichung. Wenn diese Polstellen bekannt sind, wird die Residuenrechnung zur Ermittlung der inversen Transformierten angewendet, welche die Reaktionsfaktoren enthält. Die Veröffentlichung behandelt das Verhalten der charakteristischen Gleichung und der zugehörigen Beziehungen und gibt eine verbesserte Methode zum Auffinden der Lösung an, mit welcher Reaktionsfaktoren effizient berechnet werden können.

УСОВЕРШЕНСТВОВАННАЯ ПРОЦЕДУРА ОПРЕДЕЛЕНИЯ КОРНЕЙ, ИСПОЛЬЗУЕМАЯ ПРИ РАСЧЕТЕ НЕУСТАНОВИВШЕГОСЯ ТЕПЛОВОГО ПОТОКА ЧЕРЕЗ МНОГОСЛОЙНЫЕ ПЛИТЫ

Аннотация—Так называемыми "факторами отклика" являются потоки на внутренней и внешней поверхностях одномерной многослойной плиты, обуславливаемые единичными треугольными температурными импульсами, прилагаемыми попеременно к внутренней и внешней поверхностям, причем противоположная поверхность поддерживается при постоянной температуре. Новейшие методы определения факторов отклика включают численный поиск корней характеристического уравнения для преобразованного по Лапласу решения уравнения теплопроводности. После определения полюсов с помощью метода вычетов находится обратное преобразование, которое дает факторы отклика. Проведен анализ поведения характеристического уравнения и других связанных с ним уравнений и предложен усовершенствованный метод определения корней, который позволяет эффективно рассчитывать факторы отклика.